Exercise 44

Find the critical numbers of the function.

$$f(x) = x^{-2} \ln x$$

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Solution

A critical number is a value of x for which the derivative is zero or nonexistent. Take the derivative of the function.

$$f'(x) = \frac{d}{dx}(x^{-2} \ln x)$$

$$= \left[\frac{d}{dx}(x^{-2}) \right] \ln x + x^{-2} \left[\frac{d}{dx}(\ln x) \right]$$

$$= (-2x^{-3}) \ln x + x^{-2} \left(\frac{1}{x} \right)$$

$$= -\frac{2 \ln x}{x^3} + \frac{1}{x^3}$$

$$= \frac{-2 \ln x + 1}{x^3}$$

Set what's in the numerator equal to zero, and set what's in the denominator equal to zero. Solve both equations for x.

$$-2 \ln x + 1 = 0 \qquad\qquad\qquad x^3 = 0$$

$$\ln x = \frac{1}{2} \qquad\qquad\qquad x = 0$$

$$x = e^{1/2} \qquad\qquad\qquad x = 0$$